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A NEW DIRECTION ADAPTIVE SCHEME FOR IMAGE INTERPOLATION

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ABSTRACT

In this paper, we present a novel image interpolation method based on variational models with both smoothing and orientation constraints. By introducing the orientation constraint, we simplify the nonlinear PDE problem into a series of problems with explicit solutions. In our model, the gradient directions for the interpolated pixels are first estimated using a modified orientation diffusion method. Using these estimated gradient directions adaptive directional interpolation is carried out. An effective numerical implementation of the adaptive directional interpolation is presented for the case of upsampling by factors of two. This implementation had very low complexity and is well suited for real-time applications.

1. INTRODUCTION

Image interpolation or image magnification is used to increase the resolution of an image by estimating the pixel intensities on an upsampled grid. The most commonly used image interpolation methods, such as nearest neighbor interpolation, bilinear interpolation and bicubic interpolation, can result in blocky interpolated images with staircase edges. Several schemes for edge preserving interpolation have been presented. In [1], edge positions and directions are estimated in local image areas. A truncated Fourier expansion series is used to model the shape of small patchs of an image which have edges across them. By introducing nonlinear operations high frequency components can be reconstructed. Other nonlinear methods such as the rational filter [2] can also produce better results than the traditional models. The problem of these kinds of models is that they use small regions, such as three by three, which are often not large enough for good edge extraction. In fact, a rational filter cannot distinguish a pulse from a edge, for which different interpolation schemes should be used. in [3] a Bayesian approach is employed to preserve edges and other discontinuities in image expansion. In [4], an anisotropic diffusion based model is presented for image interpolation. Another iterative interpolation model is presented in [5], in which the image interpolation problem is formulated as a inverse problem. In [6], edges are explicitly detected and two phases, rendering and correlation, are used for image interpolation in a iterative scheme. In [7], a directional adaptive interpolation method is presented which is realized by a one pass interpolation scheme.

We have found that a PDE (partial differential equation) scheme cannot be directly applied for the diffusion of enlarged grids of a image. Large scale control must be presented to guide the diffusion process. In this paper we propose a new method to solve the edge preserving image interpolation problem. By contrast to the method in [4], we have formulated the interpolation problem as a variational problem with both smoothness and orientation constraints. The reason for introducing the constraint of orientation is that we not only simplify a nonlinear PDE problem which needs iteration for its solution into a series of simple problems with explicit solution, but we also can apply more effective orientation control. By solving the variational problem we obtain the relations with which the pixels in the upsampled grid should comply. We present a one-pass solution to the image interpolation problem for upsampling by powers of two. In such cases the computational complexity of the algorithm is greatly reduced, making the method suitable for real-time applications. At the same time, we obtain results comparable to those produced by more complex iterative methods such as isophote-based interpolation [4].

2. IMAGE DIRECTION ADAPTED INTERPOLATION

The image interpolation problem can be formulated as the following variational problem,

$$\widehat{u} = \min_u\left(\int_x \int_y \rho(\|\nabla u(x,y)\|)dxdy\right) \quad (1)$$

with the constraint,

$$u(ms\Delta, ns\Delta) = u'(m,n) \qquad 0 \le m \le \left\lfloor \frac{w}{s\Delta} \right\rfloor, \quad 0 \le n \le \left\lfloor \frac{h}{s\Delta} \right\rfloor \tag{2}$$

where $u'(m,n)$ is the digital image before interpolation; $\rho(\cdot)$ is a non-negative convex function; Δ is the grid size for the upsampled digital image, assumed to be the same in both the x and the y directions; w and h are the width and height of the image, respectively; and s is the scaling factor.

Directly solving this variational problem leads to a complex and difficult computational solution, usually involving iterative operations. Another more severe problem of directly solving Equation (1) is that it is often a ill-posed problem and the solution heavily depends on the initial value selected. Therefore, the initial value would need to be very carefully selected to achieve a good image interpolation.

To mitigate this dependence on the selection of a good initial value, we introduce an additional constraint on the orientation of the interpolated image,

$$\widehat{\theta} = \min_{\theta}(\int_x \int_y \phi(\|\nabla\theta(x,y)\|)dxdy) \tag{3}$$

in which $\theta = \arg[\nabla u(x,y)]$ is the gradient angle of $u(x,y)$, and $\phi(\cdot)$ is a non-negative monotonic function. We assume that the gradient angles on the original grid are known, and that the orientations for these pixels do not change when scaling the image, i.e.,

$$\theta(ms\Delta, ns\Delta) = \theta'(m,n) \qquad 0 \le m \le \left\lfloor \frac{w}{s\Delta} \right\rfloor, \quad 0 \le n \le \left\lfloor \frac{h}{s\Delta} \right\rfloor \tag{4}$$

(In reality, we use a numerical scheme to get estimates for $\theta'(m,n)$ on the original grid.) We then simplify the nonlinear variational problem by reducing it to a series of optimization problems.

Based on these assumptions, we first solve the variational problem of (3) based on the constraint of (4), thus obtaining the estimates $\widehat{\theta}(x,y)$. Next, we solve the problem of (1) based on the constraints of both (2) and (4), thus obtaining the scaled image $\widehat{u}(x,y)$. In the next two sections, we discuss the solution of these two parts of the problem in more detail.

2.1. Orientation Estimation

In this section we discuss the problem of estimating the gradient direction of the unknown pixels on the upsampled grid. The scheme in this paper is inspired by the orientation diffusion for noisy image data presented by Perona [8]. We formulate the orientation estimation for the scaled image as the following variational problem,

$$\widehat{\theta} = \min_{\theta}(\int_x \int_y [1 - \cos(\|\bigtriangledown\theta(x,y)\|)]dxdy) \tag{5}$$

with the constraint that $\theta(ms\Delta, ns\Delta) = \theta'(m,n)$, where $\theta'(m,n) = \arg(\bigtriangledown u'(m,n))$ denotes gradient angles before upsampling and $\theta(x,y) = \arg(\bigtriangledown u(x,y))$ the gradient angle at (x,y) after upsampling. The Euler Equation corresponding to (5) is,

$$\bigtriangledown \cdot [\frac{\sin(\|\bigtriangledown\theta(x,y)\|)}{\|\bigtriangledown\theta(x,y)\|} \bigtriangledown \theta(x,y)] = 0 \tag{6}$$

In our numerical estimation, only the four neighbouring diagonal pixels are used, so that numerically Equation (6) reduces to

$$\sum_k \sin(\theta_k - \theta(i,j)) = 0 \tag{7}$$

This nonlinear equation (7) can be solved easily for its root

$$\theta(i,j) = \arctan(\frac{\sum_k \sin\theta_k}{\sum_k \cos\theta_k}) + 2k\pi \tag{8}$$

Since this orientation diffusion scheme does not account for gradient strengths, weak features may have undue influence; hence we have introduced a weighting as follows,

$$\theta(i,j) = \arctan(\frac{\sum_k A_k \sin\theta_k}{\sum_k A_k \cos\theta_k}) + 2k\pi \tag{9}$$

in which A_k is the gradient magnitude corresponding to pixel k. Thus Equation (9) becomes the numerical solution of a weighted version of Equation (5), namely,

$$\widehat{\theta} = \min_{\theta}(\int_x \int_y w(x,y)[1 - \cos(\|\bigtriangledown\theta(x,y)\|)]dxdy) \tag{10}$$

in which $w(x,y)$ is proportional to the gradient at (x,y).

2.2. Directional Interpolation Scheme

The interpolation problem is formulated as the following variational problem,

$$\widehat{u} = \min_{u}(\int_x \int_y |\nabla u(x,y)|dxdy) \tag{11}$$

with the gradient direction constraints,

$$\begin{aligned} \frac{u_x}{\sqrt{u_x^2 + u_y^2}} &= \cos[\hat{\theta}(x,y)] \\ \frac{u_y}{\sqrt{u_x^2 + u_y^2}} &= \sin[\hat{\theta}(x,y)] \end{aligned} \tag{12}$$

together with the intensity constraint of Equation (2). The Euler Equation of (11) is,

$$\nabla \cdot \left(\frac{\nabla u(x,y)}{||\nabla u(x,y)||}\right) = 0 \tag{13}$$

Expanding (13) and substituting Equation (12), we obtain, after some simplification,

$$u_{xx}\sin^2\hat{\theta} + u_{yy}\cos^2\hat{\theta} - (u_{xy}+u_{yx})\cos\hat{\theta}\sin\hat{\theta} = 0 \tag{14}$$

We discretize Equation (14) with the following scheme in which for simplicity $u(m,n)$ is used to denote $u(m\Delta, n\Delta)$:

$$u_{xx}(m\Delta,n\Delta) \simeq [u(m-1,n)+u(m+1,n)-2u(m,n)]/\Delta^2$$
$$u_{yy}(m\Delta,n\Delta) \simeq [u(m,n-1)+u(m,n+1)-2u(m,n)]/\Delta^2$$
$$u_{xy}(m\Delta,n\Delta) \simeq [u(m+1,n+1)+u(m-1,n-1) - u(m-1,n+1)-u(m+1,n-1)]/4\Delta^2$$

and

$$u(m,n-1) \simeq [u(m+1,n-1)+u(m-1,n-1)]/2$$
$$u(m,n+1) \simeq [u(m+1,n+1)+u(m-1,n+1)]/2$$
$$u(m-1,n) \simeq [u(m-1,n-1)+u(m-1,n+1)]/2$$
$$u(m+1,n) \simeq [u(m+1,n-1)+u(m+1,n+1)]/2$$

Using the above equations in (14) and solving for $u(m,n)$, we obtain the final scheme for interpolating $u(m,n)$ on the upsampled grid from its four diagonal neighbours:

$$\begin{aligned} u(m,n) = {} & \frac{1}{4}[u(m+1,n+1)+u(m-1,n+1)+ \\ & u(m+1,n-1)+u(m-1,n-1)] + \\ & \frac{1}{2}[u(m+1,n+1)+u(m-1,n-1)- \\ & u(m-1,n+1)-u(m+1,n-1)] \\ & \cos\hat{\theta}\sin\hat{\theta} \end{aligned} \tag{15}$$

Equation (15) yields a fast factor-of-two interpolation scheme as illustrated in Figure 1, in which the pixel indexing reflects the upsampled grid. The order of the interpolation process is as follows. First, pixels in the original grid are upsampled to the (m,n) grid; for example let four of these be $(m-1,n-1)$, $(m+1,n-1)$, $(m-1,n+1)$ and $(m+1,n+1)$. Then the pixel at (m,n) is interpolated from its estimated orientation and the intensities of these four diagonal neighbours. The pixels at $(m-2,n)$, $(m,n-2)$, $(m+2,n)$ and $(m,n+2)$ are interpolated in a similar way. Then, the pixel at, say, $(m-1,n)$ can be interpolated from the pixels at $(m-2,n)$, $(m-1,n-1)$, (m,n) and $(m-1,n+1)$ with the same interpolation scheme, since the relative positions of these pixels are in fact just a 45 degree rotation from the first scheme. Similarly we can obtain the interpolated pixels at $(m,n-1)$, $(m+1,n)$ and $(m,n+1)$. The process can be iterated with the resolution increased by a factor of two on each iteration.

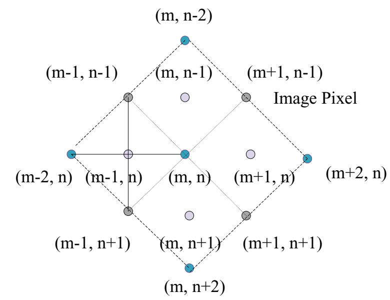

Figure 1: Fast Interpolation Scheme

3. EXPERIMENTAL RESULTS

Figure 4 shows the results of the proposed method compared with the results of two standard interpolation schemes, the bicubic and the bilinear interpolators, in Figures 2 and 3, respectively. This example uses a colour image of a flower which has been upsampled by four times in these three figures. To interpolate a colour image, we adopt a simple scheme which interpolates the R, G and B channels separately and then combines them together to produce the final interpolated result.

We can see that the proposed model is superior to the traditional bilinear and bicubic model, producing smoother interpolations over linear structures such as the stamens of the flower. By contrast, both the bicubic and the bilinear interpolators produce noticably more jagged results. The method we propose produces more pleasing results and does so efficiently and quickly.

Figure 2: Interpolation result of bicubic method.

4. CONCLUSION

The contribution of this paper is a new PDE based scheme for image interpolation. We formulate interpolation as a variational problem with both smoothness and orientation constraints. Introducing the orientation constraint allows not only the simplification of a nonlinear PDE problem needing iteration into a series of simple problems with explicit solution, but also allows more effective orientation control. We present a one-pass solution to the image interpolation problem for scaling by powers of two. The computational complexity of the algorithm is greatly reduced in this case, making the method suitable for real-time applications. At the same time, we obtain results comparable to those of more complex iterative methods.

Figure 3: Interpolation result of bilinear method.

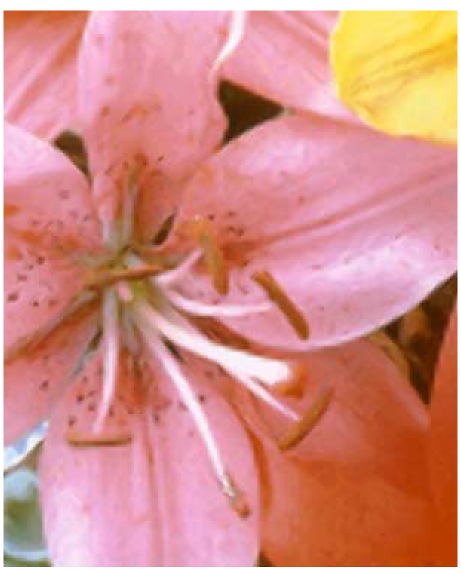

Figure 4: Interpolation result of proposed method.

5. REFERENCES

[1] K. Jensen and D. Anastassion, "Subpixel edge localization and the interpolation of still images", *IEEE Transactions on Image Processing*, vol. 4, pp. 285-295, March 1995. 1

[2] S. Carrato, G. Ramponi, and S. Marsi, "A simple edge-sensitive image interpolation filter", *Proceedings of ICIP*, Lausanne, Switzerland, 1996, pp. 711–714. 1

[3] R.S. Schultz and R.L. Stevenson, "A Bayesian approach to image expansion for improved defintion" *IEEE Transactions on Image Processing*, vol. 3, pp. 233–242, May 1994. 1

[4] B.S. Morse and D. Schwartzwald, "Isophote-based interpolation", *Proceedings of ICIP*, Chicago, Illinois, 1998. 1

[5] K. Ratakondo and N. Ahuja, "POCS based adaptive image magnification", *Proceedings of ICIP*, Chicago, Illinois, 1998, pp. 203-207. 1

[6] J. Allebach and P.W. Wong, "Edge directed interpolation", *Proceedings of ICIP*, Lausanne, Switzerland, 1996, pp. 707–709. 1

[7] X. Li and M.T. Orchard, "New edge directed interpolation", *Proceedings of ICIP*, Vancouver, Canada, 2000. 1

[8] P. Perona, "Orientation diffusions", *IEEE Transactions on Image Processing*, vol. 7, pp. 457–467, March 1998. 2