Predicting Cone Quantum Catches under Illuminant Change

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Abstract

Given LMS cone quantum catches from a surface under a first illuminant what is the best method of predicting what the corresponding quantum catches will be for the same surface under a second illuminant given only the quantum catches of a white surface under both illuminants? The von Kries rule is one well known method. In this paper, two new prediction methods along with a variation on an existing third method are introduced and then compared experimentally. In contrast to the von Kries rule which is equivalent to a diagonal transformation, all three methods estimate a full 3-by-3 linear transformation mapping LMS values between illuminants. All the new methods perform better than the von Kries rule.

Introduction

When there is a change in illumination, the cones' quantum catches change. We address the problem of predicting how they change. In particular, we are concerned with predicting the LMS cone signals under a second illuminant given the LMS cone signals under a first illuminant along with the LMS cone signals of a white surface under each of the illuminants.

Although the problem we address here of predicting cone quantum catches may be relevant to chromatic adaptation, it is not the same as chromatic adaptation. Models of chromatic adaptation [7,8] aim to predict which colors have corresponding appearances to a human subject under a change in illuminant. Accurate prediction of cone quantum catches will not necessarily lead to accurate appearance matches; however, it might be useful as a sub-component of a full color appearance model.

One common method of predicting LMS under a second illuminant is the von Kries rule[6], which involves a diagonal model of illuminant change. Given the LMS quantum catch x_a observed for a surface under illuminant a the diagonal model predicts the corresponding LMS quantum catch of the same surface under illuminant b as:

$$x_b = Dx_a$$

where D is a 3-by-3 diagonal matrix.

The diagonal model is limited to 3-parameters, We would prefer to use the more general full 9-parameter, 3x3 linear model of the form:

$$x_b = Mx_a \qquad (1)$$

where M is 3-by-3.

One reason the full 3x3 linear model is not used is that usually there is not enough information available to determine the 9 coefficients directly, especially if all we know about the two illuminants are the LMS values of a white surface under each illuminant.

The question we address here is: Are there other non-diagonal models that would perform better than the diagonal model? We answer the question with 3 methods we will call the Lighting-Matrix Estimation Method (LME), the Palette Method (PM) and the Characteristic Vector Method (CVM). PM is entirely new. LME is closely related to a derivation by Maloney [10]. CVM is a modification of an earlier PCA-based method [2].

LME employs the assumptions that illuminant spectra and reflectance spectra are approximated well by 3-dimensional linear models along with the machinery of Maloney and Wandell's "lighting matrix"[9]. As will be shown below, the restriction to 3 dimensions means that LMSs from the 2 whites provide enough information to solve for the 3-by-3 lighting matrix mapping from one 3D illuminant to the other. The method starts from the fact that given the LMS of a known reflectance (white in this case), a 3D model of the illuminant spectrum can be calculated. Then, based on a 3D model of illumination, the transform mapping one lighting matrix to another is calculated.

PM involves an analysis of the space of possible palettes of LMS signals that occur under different illuminants. We define an illuminant's palette as the set of all LMS signals obtained from a training set of surface reflectances under that illuminant. We use the term 'palette' rather than 'gamut' to avoid confusion with the

similar, but different, us of the term 'gamut' in the context of gamut mapping algorithms. Characteristic Vector Analysis is used to extract a 3-dimensional linear model approximating the set of possible palettes. A change in illumination causes a change in observed palette.

CVM considers the 9-dimensional space of 3x3 transformations that model illuminant change and then finds the 3-dimensional subspace that best approximates these matrices. The required illuminant transformation matrix is then built up from the 3 basis matrices based on the LMS of white.

LME: Lighting-Matrix Estimation Method

Suppose finitely sampled illuminant spectra and surface reflectances are modeled by 3-dimensional linear models:

$$E(\lambda_n) \approx \sum_i^3 E_j(\lambda_n)$$

$$S(\lambda_n) \approx \sum_j^3 S_j(\lambda_n)$$

Let the cone sensitivity functions be $R_k(\lambda_n) \quad k = 1 \ldots 3.$ Following Wandell[14] and Maloney[10], we can construct a lighting matrix Λ mapping surface reflectance weights σ to LMSs:

$$l = \Lambda\sigma$$

where the *kj*th entry of Λ is

$$\sum_{n=1}^{N} E(\lambda_n) S_j(\lambda_n) R_k(\lambda_n) \Delta\lambda_n$$

For each of the illuminant basis vectors, E_i, we similarly define Λ_e e=1, 2,3. Using the linear model for $E(\lambda_n)$, we can also express Λ as a weighted combination of the Λ_e:

$$\Lambda = \varepsilon_1\Lambda_1 + \varepsilon_2\Lambda_2 + \varepsilon_3\Lambda_3$$

In addition, for a given surface reflectance $S(\lambda_n)$ define the surface matrix Ω^s with *ki*th entry:

$$\sum_{n=1}^{N} S(\lambda_n) E_i(\lambda_n) R_k(\lambda_n) \Delta\lambda_n$$

We can write the LMS response vector l as a linear transform from the ε weights describing an illuminant:

$$l = \Omega^s \varepsilon$$

Then given LMS, l_a of white under illuminant *a*, calculate the weights ε^a describing that illuminant via

$$\varepsilon^a = [\Omega^w]^{-1} l_a$$

where Ω^w is constructed with a surface reflectance function representing white (e.g., $W(\lambda_n) = 1$).

The lighting matrix for *a* can then be calculated as:

$$\Lambda^a = \varepsilon_1^a\Lambda_1 + \varepsilon_2^a\Lambda_2 + \varepsilon_3^a\Lambda_3$$

Similarly, from LMS, l_b of white under illuminant *b* we obtain

$$\varepsilon^b = [\Omega^w]^{-1} l_b$$

and

$$\Lambda^b = \varepsilon_1^b\Lambda_1 + \varepsilon_2^b\Lambda_2 + \varepsilon_3^b\Lambda_3$$

From the LMS t_a of a surface *T* under light *a* we obtain its reflectance σ as:

$$\sigma = [\Lambda^a]^{-1} t_a$$

The LMS t_b of reflectance σ illuminated by *b* is then

$$t_b = \Lambda^b\sigma$$

This derivation parallels that of Maloney[10]. Taking it a step or two further, we can see it can be used to express a type of chromatic adaption transform. In particular, we have

$$t_b = \Lambda^b[\Lambda^a]^{-1} t_a$$

Letting $M = \Lambda^b[\Lambda^a]^{-1}$,

$$t_b = M t_a$$

M is the desired 3x3 matrix that transforms the LMS quantum catch of a surface under illuminant a to its LMS quantum catch under illuminant b.

We note that if a von Kries model of illumination change holds then the transform matrix M that is calculated is a diagonal matrix (or can be made to be diagonal through an appropriate change of sensor basis[3]). Thus, LME method can be seen to generalize the von Kries approach.

PM: Illumination Palette Method

Consider n uniformly sampled surface reflectances represented as *n* column vectors $T_i \; i = 1 \cdots n$ along with a m similarly sampled illuminant spectra $F_j \; j = 1 \cdots m$.

For surface Ti under illuminant Fj the LMS quantum catch vector is $c_{i,j} = [c^1_{i,j}, c^2_{i,j}, c^3_{i,j}]^T$ where

$$c^k_{i,j} = \sum_{n=1}^{N} F_j(\lambda_n) T_i(\lambda_n) R_k(\lambda_n) \Delta\lambda_n$$

Define an illumination palette G_i to be a $3Nx1$ matrix the rows of which contain the components of the LMS responses arising from all reflectances under illuminant *i*

$$G_i = [c^T_{i,1}, ..., c^T_{i,j}, ..., c^T_{i,n}]$$

Then define the world palette, Γ, as the $3nxm$ matrix whose columns consist of all the illumination palettes G_i

$$\Gamma = \left\lfloor G_1^T \quad G_2^T \quad ... \quad G_m^T \right\rfloor$$

Each illumination palette is a point in this high-dimensional space. A move from one point in this space to another represents a change in illumination. Since the illumination palettes are all related to one another by the common set of reflectances, we expect the underlying dimensionality of Γ to be low. In fact, if the von Kries model were to hold exactly, it would have dimension 3.

By singular value decomposition of Γ, we extract a 3-dimensional basis approximating the column vectors of Γ.

The remaining issue is to find the mapping between illuminants. This is accomplished by solving for the illumination palettes to which the whites belong. Without loss of generality, we specify the first reflectance to be ideal white, i.e.,

$[1, \cdots, 1]^T$.

Assume the LMS 3-vectors of white under the two illuminants are l_a and l_b.

Each Γ_i basis palette is a $3Nx1$ matrix containing N LMS triples stretched out as a vector. Let us reshape this into a table of LMSs and denote the resulting $Nx3$ matrix as Q_i. By assumption, the first row of Q_i is an LMS response to a white surface. Since we have three $Q_i \; i = 1 \cdots 3$, we correspondingly have 3 white responses $e_i \; i = 1 \cdots 3$. At this point, it is a simple matter to generate the palette corresponding to the white response under illuminants *a*. We have a set of 3 equations in 3 unknowns which is first solved:

$$\alpha = [e_1, e_2, e_3]^{-1} l_a$$

The whole palette is then

$$Q_a = \alpha_1 Q_1 + \alpha_2 Q_2 + \alpha_3 Q_3$$

Similarly, we obtain Q_b from l_b.

The transform between palettes, and hence illuminants, is then

$$T = Q_a^+ Q_b$$

where "+" denotes the pseudo-inverse.

Given any LMS 3-vector x_a under illumination a, the predicted LMS 3-vector x_b under illumination b is

$$x_b = T x_a$$

CVM: Characteristic Vector Method

The space of illumination transformation matrices M from Eqn. (1) is 9 dimensional. However, in [2] it was shown that the underlying dimensionality of matrices M was close to 3 when the illuminants were normalize to unit energy and the mean of the matrices was removed. These last two restrictions are removed here.

To determine the dimensionality of the space of matrices M, we first construct a large set of corresponding LMSs under different pairs of illuminant. These pairs are formed from the 140 illuminants in the Simon Fraser University database [1]. For each illuminant pair, the corresponding LMSs for surface reflectances from the Kodak reflectance and Krinov databases [5,13] are calculated. The best, in the least-squares sense, 3-by-3

illumination transformation matrix, M, mapping one set to the other is then determined. For n illuminant pairs, we obtain n new such matrices M.

Let us represent the set of all transform matrices M in a 9xN matrix A (each column of A is a 3x3 matrix stretched out as a vector). It is well known that if we wish to find the set of 3 9x1 vectors which is the best basis for describing A we use the singular value decomposition:

$$A = UDV^T$$

U and V are Nx9 orthogonal matrices and D is a 9x9 diagonal matrix. The first 3 columns of U form the optimal basis in that linear combinations of these columns best approximate A in a least squares sense. The first 3 columns of U are sometimes called the first 3 characteristic vectors of A. Characteristic vector analysis in effect carries out principal components analysis when the mean is not subtracted.

This produces characteristic vectors u_i which can be reshaped back into 3x3 matrices U_i. An illumination transformation matrix M can then be approximated as

$$M \approx \sum_{i=1}^{9} \eta_i U_i ,$$

where $\eta_i = m \cdot u_i$.

The remaining issue is how to use this model for LMS prediction. Given the LMS 3-vectors, l_a and l_b, of white under the two illuminants, the coefficients c_i required to predict LMSs under illumination b from LMSs from corresponding reflectances illumination a can be determined as follows. Since $l_b = l_a M$, we have

$$\begin{aligned} l_b &= c_1 l_a U_1 + c_2 l_a U_2 + c_3 l_a U_3 \\ &= c_1 [l_a U_1(1), l_a U_1(2), l_a U_1(3)] \\ &\quad + c_2 [l_a U_2(1), l_a U_2(2), l_a U_2(3)] \\ &\quad + c_3 [l_a U_3(1), l_a U_3(2), l_a U_3(3)] \\ &= [c_1, c_2, c_3] Q \end{aligned}$$

with

$$Q = \begin{bmatrix} l_a U_1(1) & l_a U_1(2) & l_a U_1(3) \\ l_a U_2(1) & l_a U_2(2) & l_a U_2(3) \\ l_a U_3(1) & l_a U_3(2) & l_a U_3(3) \end{bmatrix}$$

$U_i(j)$ denotes column j of matrix U_i.

Letting $c = [c_1, c_2, c_3]^T$,

$$c = l_b Q^{-1}$$

Let k_a be the LMS 3-vector of a surface under illuminant a. The corresponding LMS under illuminant b then is predicted as

$$k_b = M k_a$$

Experimental Results

To test the prediction methods, we use a subset of the illuminants and reflectances in the database as a training set and the remainder as a test set. The corresponding LMS values are calculated based on the cone sensitivity functions[12]. We use 100 different illuminants and 500 different reflectances including white in the training process. The test set consists of 20,040 LMS values based on all combinations of the remaining 501 reflectances and 40 illuminants in the database. The average error in predicting LMS is tabulated in Table 1 for each of the methods. The row labeled "ideal 3x3" refers to the best possible 3x3 linear transformation based on solving for the optimal transform in terms of a mean square error fit over the testing data. Ideal 3x3 provides a lower bound on the error that can be achieved using a linear model.

	L	M	S	Avg CIE ΔE	ΔE Std. Dev.	Median ΔE
LME	0.023	0.028	0.024	3.65	3.02	1.93
PM	0.017	0.026	0.028	3.09	2.40	1.72
CVM	0.017	0.026	0.021	2.98	2.40	1.64
Von-Kries	0.033	0.043	0.011	3.48	2.78	2.31
Ideal 3x3	0.012	0.015	0.008	0.98	1.08	1.05

Table 1. Mean, standard deviation and median CIELAB ΔE error for each method as well as the relative errors for each cone channel. The training and testing sets are disjoint.

Reversibility and Preserving Whites

On looking at the results in Table 1 one might be inclined to favor CVM since it gives the lowest errors (with the exception of the ideal 3x3 matrix which we cannot

solve for given only the 2 white points). However, a disadvantage of CVM is that it is not reversible. That is to say if we take data under D65 and map it to A and then back again we do not arrive at the same starting point. Indeed, even the white points may change. Moreover, this problem becomes worse the more data is manipulated. If we shuttle back and forth between D65 and A our data moves further from where it should be (under D65).

Similarly for PM, it is easy to show that whites need not be mapped correctly across illuminants. This of course could be fixed by adopting a regression formula to enforce the correct mapping of white[4]. However, this fix does not make the method reversible for other colors. The LMS for a red surface mapped from D65 to A and back again will still not end up with its original LMS value.

In the case of, LME where both light and surface are 3 dimensional, reversibility is ensured. Thus, it is possible that in a color management application, we might choose the LME approach even though it has a higher error because this error is bounded and does not increase with the number of times we map data between illuminants. The method may also be improved by employing 2-mode analysis[11] in the construction of the illuminant and reflectance basis functions. This will be tested in future work.

Conclusion

This paper introduced and tested three different methods for predicting LMS cone quantum catches under a change in illuminant. In contrast to von Kries rule diagonal models, the new methods compute full 3x3 linear transformations All the methods outperform the diagonal model. Despite its higher average error, two advantages of the Lighting-Matrix Estimation method (LME) are that it preserves white and is reversible.

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